Directorate of Distance Education

T.D.C. VIth Semester Examination 2015 (2012-15)

Subject: Mathematics

Paper : 7th

Model Paper (Full Marks – 80)

Answer any five questions, selecting at least one from each group.

Group – A

1.a) State and Prove the theorem of Parallel axes on moment of inertia

Or

State and prove theorem of perpendicular axes on moment of intertia.

b) Find the moment of interia of a uniform rod of length 2a and mass M about an axis through the mid point and perpendicular to it.

Or

Find the M.I. of uniform rectangular lamuia of mass M and sides 2a and 2b about an axis through its centre (i) Parallel to side 2a and (ii) Parallel to side 2b.

Or

Find the moment of inertia of a right circular cylinder about its axis.

2.a) Explain what do you mean by momental ellipsord. Prove that for every body there exists at every point O, a set of three mutually perpendicular axes such that the products of inertia of the body about them taken two at a time vanish.

Or

Discuss the equimomental system and establish the necessary and sufficient conditions for two systems to be equimomental.

b) Show that the momental ellipsoid at the centre of an ellipsoid 1s

$$(b^2 + c^2)x^2 + (c^2 + a^2)y^2 + (a^2 + b^2)z^2 = \text{Constant}$$

Or

Show that the moment and product of inertia of a uniform triangle about any time are the same as the moments and product of inertia of three particles placed at the middle points of the sides each equal to one third of the mass of the triangle.

3.a) State D'Alemberts Principle and obtain the general equation of motion of a riged body in three dimensions under the action of given external forces.

Or

Find the equation of motion is two dimensions when the forces acting on the body are finite.

Or

When the body is moving in two dimensions then express the kinetic energy in terms of the motion of the centre of inertia and motion relative to centre of inertia.

Or

Find the moment of momentum of the body about the fixed origin O where the body is moving in two dimensions.

b) A plank of mass M is initially at rest along a base of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M′, standing from the upper end walks down the plank so that it does not move, show that he gets to the other end in time

$$\sqrt{\left\{\left[\frac{2M'a}{(M+M')gsin\alpha}\right]\right\}}$$

Where a is the length of the plank

Or

A uniform rod OA of length 2a, free to turn about its end O, revolved with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either zero or $Cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$

4.a) Define a compound pendulum and find the time of a complete oscillation of a compound pendulum .

Or

Define centre of suspension and centre of oscillation and show that both centre of suspension and centre of oscillation are convertible.

b) A solid homogeneous cone of height h and vertical angle 2α oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{1}{3}h(4+tan^2\alpha)$

Or

A bent lever whose arms are of length a and b, the angle between then being α, makes small oscillation in its own plane about the fulcrum, show that the length of the corresponding simple equivalent pendulum is

$$\frac{2}{3}\quad\frac{a^3+b^3}{\sqrt{\{(a^4+2a^2b^2cos^2\alpha+b^4)\}}}$$

Group – B

5.a) Prove that the attraction of an infinite rod at an external point varies inversely as the distance of the point from the rod.

Or

Find the attraction of a uniform thin circular disc, of radius a and small thickness k, at a point P on the axis of the disc at a distance p from the centre.

Or

Find the attraction of a then uniform spherical shall on an external and internal point P.

b) Find the potential of a circular disc of radius a, small thickness k and density ρ, at an external point P an its axis distant p from the centre.

Or

Find the potential of a thin uniform spherical shell at any point (internal or external).

Or

Find the potential of a solid sphere, of mass M and radius a at any point (internal or external).

6.a) State and Prove Poisson's theorem.

Or

Define equipotential surface and find the condition that a family of surfaces be a possible family of equipotential surface in free space.

b) Show that a family of right circular cone with common axis and vertex is a possible family of equipotential surfaces and find the potential function.

Or

Show that the system of co-axial cylinders

$$x^2 + y^2 + 2\lambda x + c^2 = 0$$

can form a system of equipotential surfaces, and find the law of potential.

Or

Show that

$$(x - c)^2 + y^2 = \lambda\{(x + c)^2 + y^2\}$$

represents a family of equipotential surfaces and that

$$V = A \log \lambda + B$$

Where V is the potential function and A and B are arbitrary constants.

7.a) Prove that the pressure of a homogeneous liquid at all points in the same horizontal plane is the same.

Or

If a fluid, homogeneous or heterogeneous, be at rest under the action of gravity, the pressure at all points in a horizontal plane are the same.

b) Find the centre of pressure of a parallelogram immersed in a homogeneous liquid with one side in the free surface.

Or

Find the C.P. of a triangular area immensed in a homogeneous liquid with its vertex in the surface and base horizontal.

Or

Find the C.P. of a triangular area immerged in a homogeneous liquid with one side in the surface of the liquid and vertex downward.

8.a) Discuss the general condition of equilibrium of a body floating freely in a homogeneous liquid.

Or

Discuss the condition of equilibrium of a body floating freely in two liquids that do not mix.

b) A substance, whose density is ρ, is weighed by means of weights of which the density is ρ', if σ be the density of the atmospheric air, find the true weight of the substance corresponding to any apparent weight.

Or

State and explain the principle of Archimedes in connection with a body immersed wholly or partially in a fluid.

Or

Show that right circular cone of density ρ and semi vertical angle α can float vertex downwards in liquid of density σ with one generator vertical and base just clear of the fluid if

$$\rho = \sigma (\cos\alpha)^{3/2}$$

Group – C

9. Solve any two of the following

i). $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (1 - \cot x)y = e^x \sin x$

ii). $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + (1 - x)y = x^2e^{-x}$

iii). $x^2y'' - (x^2 + 2x)y' + (x + 2)y = x^3e^x$

iv). $x^2y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$

v). Solve by method of variation of parameters

$y_2 + n^2y = \text{Sinx}$

$y_2 + a^2y = \text{Cosecax}$

$y_2 + 4y = 4\tan 2x$

10.a) Find the necessary and sufficient condition for the integrability of the equation

$$P\,dx + Qdy + R\,dz = 0$$

Where P, Q , R are functions of x, y, z

b). Solve

i). $(yz + xyz)\,dx + (zx + xyz)dy + (xy + xyz)dz = 0$

ii). $yz \log zdx – zx \log z\,dy + xy\,dz = 0$

iii). $(yz + z^2)dx – xzdy + xydz = 0$

iv). $xz^3dz – zdy + 2ydz = 0$

v). $3x^2dx + 3y^2dy – (x^3 + y^3 + e^{2z})\,dz = 0$

vi). $(2x^2 + 2xy + 2xz^2 + 1)\,dx + dy + 2zdz = 0$

vii). $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$

viii). $\frac{adx}{(b-c)yz} = \frac{bdy}{(c-a)zx} + \frac{cdz}{(a-b)xy}$

11.a) Discuss Charpit's method for solution of equations $f(x, y, z, p, q) = 0$

b) Solve

i) $px + qy = pq$

ii) $z^2(p^2z^2 + q^2) = 1$

iii) $yzp^2 - q = 0$

iv) $(p^2 + q^2)\,y = qz$

v) $(p^2 + q^2)\,x = pz$

vi) $p^2x + q^2y = z$

12) Discuss Monge's method to solve the differential equn

$$Rr + Ss + Tt = V$$

b) Solve

i) $r = a^2t$

ii) $r - t\cos^2x + p\tan x = 0$

iii) $t - r\sec^4y = 2q\tan y$

iv) $pt - qs = q^3$

v) $x^2r - 2xs + t + q = 0$

vi) $(x - y)(xr - xs - ys + yt) = (x + y)(p - q)$

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Directorate of Distance Education

T.D.C. VIth Semester Examination 2015 (2012-15)

Subject: Mathematics (Hons)

Paper : 8th (Numerical Analysis)

Model Paper (Full Marks – 80)

Answer any five questions

Group – A

1. What do you mean by operators E and Δ. Discuss the algebraic properties of operators E and Δ. Also show that $E = 1 + \Delta$

2. Evaluate

   i) $\Delta (x^2 + 2x)$ ii) $\Delta \log 2$

   iii) $\Delta \sin x \cos 3x$ iv) $\Delta e^{ax} \log bx$

   v) $\Delta \tan^{-1} x$

3. Evaluate

   i) $E^3 e^x$ ii) $E^n e^x$

   iii) $(E - 1)(E - 2)x$ iv) $(E - 2)(E - 1)3^x$

   v) $E^2 x^2$

4. Discuss Bisection or Regula-Falsi method to find the roots of algebraic or transcendental equations.

5. Find the roots of the equation $x^3 - x - 1 = 0$ lying between 1 and 2 by bisection method.

6. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by method of false position upto three places of decimals.

7. Discuss Newton – Raphson's method to find the roots of algebraic or transcendental equations.

8. Find the real roots of the equation $3x = \cos x + 1$ by Newton – Raphson's method.

9. Obtain Newton-Gregory's formula for forward/Backward interpolation formula with equal intervals.

10. From the following table, find the number of students who obtain less than 45 marks.

Marks	No. of students
30 – 40	31
40 – 50	42
50 – 60	51
60 – 70	35
70 – 80	31

11. The population of a town was as given. Estimate the population for the year 1925.

Year (x)	1891	1901	1911	1921	1931
Population in lakh (y)	46	66	81	93	101

12. Derive formula to find derivatives using Newton's forward/backward difference formula.

13. Given that

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find dy/dx at x = 1.1

13. Establish

a) General quadrature formula

b) Trapezoidal Rule

c) Simpson's 1/3rd and 3/8th Rule

d) Weddle's rule

14. Find the value of the integral

a) $\int_0^1 \frac{dx}{1+x^2} dx$ using Simpson's 1/3rd and 3/8th rule.

b) $\int_0^1 \frac{1}{1+x} dx$ using trapezoidal rule Simpson's 1/3rd and 3/8th rule.

15. Deduce Picard's method to find the approximate solution of ordinary differential equation.

16. Using Picard's method to obtain the solution upto third approximation of the differential equation

i) $dy/dx = x + y^2$, given that $y(0) = 0$

ii) $dy/dx = x + y$, given that $y(0) = 1$

17. Apply Runge-Kutta method to find an approximate value of y for x = 0.2 in steps of 0.1 if $dy/dx = x + y^2$, given that y = 1 when x = 0.

18. Using Runge-Kutta method to find an approximate value of y for x = 0.2

If $\frac{dy}{dx} = \frac{y-x}{y+x}$ given that y(0) = 1 Take h = 0.2

19. Solve the following simultaneous linear equations using Gauss's elimination method

(i) $x + y + 9z = 15$

$x + 17y - 2z = 48$

$30x - 2y + 3z = 75$

(ii) $2x_1 + 4x_2 + x_3 = 3$

$3x_1 + 2x_2 - 2x_3 = 2$

$x_1 - x_2 + x_3 = 6$

20. Solve the following simultaneous linear equations using Jordon's method

(i) $x + 2y + z = 8$

$2x + 3y + 4z = 20$

$4x + 3y + 2z = 16$

(ii) $x + y + z = 9$

$2x - 3y + 4z = 13$

$3x + 4y + 5z = 40$

21. Solve the following system of equations by Gauss – Seidel method

(i) $10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10z = 22$

(ii) $27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

22. Solve by Relaxation method, the equations

(i) $9x - 2z + z = 50$

$x + 5y - 3z = 18$

$-2x + 2y + 7z = 19$

(ii) $10x - 2y - 2z = 6$

$-x + 10y - 2z = 7$

$-x - y + 10z = 8$

23. Apply Graeffels root squaring method to determinante the approximate roots of the equation

$$x^3 - 3x^2 - 6x + 8 = 0$$

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