Babasaheb Bhimrao Ambedkar Bihar University, Muzaffarpur
Directorate of Distance Education
T.D.C. 2nd Semester Examination 2016 (Session 2015-18)
Subject:- Mathematics (Hons.)
Paper – 2nd
Model Paper (Full Marks – 80)

Answer any four questions:-
किन्हीं चार प्रश्नों के उत्तर दें।

1. State and prove De Moivre's theorem.
2. Find the expansion of $\sin x$ in ascending powers of x.
3. Find the expansion of $\cos x$ in ascending powers of x.
4. State and prove Gregory's theorem.
5. Find the sum of sines of n angles in A.P.
6. Find the sum of cosines of n angle in A.P.
7. Solve $x^7 - 1 = 0$
8. If $\tan x = n \tan y$ find a series for x.
9. Prove that nth roots of unity forms a series in G.P.
10. Solve $x^5 - 1 = 0$
11. If $(1+x)^n = p_0 + p_1 x + p_2 x^2 + \ldots$ then prove that

$$p_0 - p_2 + p_4 - \ldots = 2^{\frac{n}{2}} \cos \frac{n\pi}{4}$$

and $$p_1 - p_3 + p_5 - \ldots = 2^{\frac{n}{2}} \sin \frac{n\pi}{4}$$

(2)

(12) Prove that
$$(1-i)^n+(1+i)^n = 2^{\frac{n}{2}} \cos\frac{n\pi}{4}$$

(13) If
$$x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$$
then prove that
$$x_1 \cdot x_2 \cdot x_3 \ldots \infty = -1$$

(14) If $(a_1+ib_1)(a_2+ib_2)\ldots(a_n+ib_n) = A+iB$

then prove that

(i) $\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} \ldots + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$

(ii) $(a_1^2+b_1^2)(a_2^2+b_2^2)\ldots(a_n^2+b_n^2) = A^2+B^2$

(15) Expand $(1-2x\cos\theta+x^2)$ in a series of powers of x when $|x|<1$

(16) Using DeMoivre's theorem solve $x^7+x^5+x^3+1=0$

(17) Sum the series
$$1+c\cos\alpha+c^2\cos 2\alpha + c^3\cos 3\alpha + \ldots + n \text{ terms}$$

(18) Sum the series
$$\sin\alpha+\frac{1}{2}\sin 2\alpha+\frac{1}{2^2}\sin 3\alpha + \ldots$$

(19) Sum the series
$$1+\frac{1}{2}\cos 2\theta+\frac{1}{2\cdot 4}\cos 4\theta + \frac{1\cdot 3}{2\cdot 4\cdot 6}\cos 6\theta \ldots \infty$$
when $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(2I) Sum the series

$$1 + \frac{\cos 40}{4} + \frac{\cos 80}{L} \ldots \infty$$

L.P.P

(1) Explain general and matrix formulation of L.P.P

(II) Define convex set. Prove that intersection of two convex sets is a convex set

(III) Prove that a hyperplane is a convex set

(IV) Prove that a sphere is a convex set.

(5) Prove that the set of feasible solution of L.P.P is a convex set

(6) Solve graphically the following L.P.P

maximize $z = 3x_1 + 5x_2$

Subject to

$x_1 + 2x_2 \leq 2000$

$x_1 + x_2 \leq 1500$

$x_2 \leq 600$

$x_1 \geq 0, \; x_2 \geq 0$

(3) Solve graphically the following L.P.P.

Minimise $z = x_1 + x_2$

Subject to constraints

$$x_1 + 2x_2 \le 20$$

$$x_1 + x_2 \le 15$$

$$x_1 \le 6$$

$$x_1, x_2 \geq 0$$

Theory of equation

(1) State and prove fundamental theorem of algebra

(ii) Find the condition that the roots of the equation

$$x^4 + px^3 + qx^2 + rx + s = 0$$

should be in H.P.

(iii) Find the condition that the roots of the equation of the biquadratic

$$x^4 + px^3 + qx^2 + rx + s = 0$$

may be in G.P.

(4) If α, β and γ be the roots of the equation $x^3 + px^2 + qx + r = 0$

then calculate

$$(\alpha^2+1)(\beta^2+1)(\gamma^2+1)$$

(5) If α, β, γ be the roots of the cubic $x^3 + px^2 + qx + r = 0$, find the value of the symmetric functions $\sum \frac{\beta}{\gamma} + \frac{\gamma}{\beta}$

(6) If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$ find the value of

$$\sum \frac{\beta^2+\gamma^2}{\beta+\gamma}.$$

(7) Solve the equation $x^3-7x^2+36=0$ given that one root is double of another root.

(8) Prove that in equation with real coefficients irrational roots occur conjugate pairs.

(9) Prove that every equation of nth degree has n roots and no more.