Babasaheb Bhimrao Ambedkar Bihar University, Muzaffarpur
Directorate of Distance Education
P.G. 3rd Semester Examination 2015 (Session 2014-16)
Subject:- Mathematics
Paper – 9th
Model Paper (Full Marks – 80)

Numerical Analysis

(1) What do you mean by difference of zero?
Calculate the value of
$\Delta^2 0^5, \Delta^2 0^8, \Delta^4 0^5$

(2) Obtain Newton's forward difference formula for equal intervals.

(3) State and prove Fundamental Theorem on difference calculus.

(4) Find the relation between operator E of finite difference and differential coefficient D of differential calculus.

(5) Define factorial notation. Prove that
$\Delta^n x^{(n)} = \lfloor n h^n$
and $\Delta^{n+1} x^{(n)} = 0$

(6) Prove the recurrence relation
$\Delta^n 0^m = n[\Delta^{n-1} 0^{m-1} + \Delta^n 0^{m-1}]$

(7) Obtain Newton's forward difference formula for equal intervals.

(8)

(8) Define Leibnitz's rule. Use Leibnitz's rule for finding $\Delta^3(x^2 a^x)$ if interval of differencing is h.

9. Find ~~the~~ and correct by means of differences the error in the following table.

20736, 28561, 38416, 50625, 65540, 83521, 104976, 130321, 160000.

(10) Discuss errors and their analysis

(11) Represent the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and its successive differences into factorial notation.

(12) Find the function whose first-difference is $9x^2 + 11x + 5$

(13) Discuss Newton-Raphson Method

(14) Write comparison of Newton's method with the Regula-Falsi method

(15) Discuss Graeffe's root squaring method

(16) Apply Graeffe's method to find all the root of the equation

$x^4 - 3x + 1 = 0$

(17) obtain $\sqrt{12}$ to five places of decimals by Newton's Raphson method

(18) Find the cube roots of 10

(19) Discuss the bisection method formula.

(20) Discuss the method of false position.

(21) Discuss the iteration method

(22) Write the Newton– Gregory
~~formula for~~ ~~inter~~ interpolation

(25) write ~~new~~ Newton – Gregory
~~formula~~ for Backward interpolation.

(26) given $\sin 45^\circ = .7071$, $\sin 50 = .7660$, $\sin 60^\circ = .8660$, $\sin 55^\circ = .8192$
find $\sin 52^\circ$ by using any method of interpolation.

(27) Define divided difference.
Divided difference are symmetric functions of their arguments i.e. the value of any divided difference ~~divided~~ is independent of the order of the argument. Prove this

(23) The n^{th} divided difference of a polynomial of the n^{th} degree are constant. Prove this

(24) Show that the n^{th} divided difference can be expressed as the quotient of two determinants each of order $n+1$.

(25) The n^{th} divided difference can be expressed as the product of multiple integrals. Prove this

(26) Discuss Newton's formula for unequal intervals

(27) Define Lagrange's Interpolation formula for unequal intervals

(28) Obtain Newton's backward difference formula

(29) Using Newton's forward difference formula, find the sum

$S_n = 1^3 + 2^3 + 3^3 + \dots + n^3$

(30) Define Gauss's Interpolation formula

(31) Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \ln x$ defined by the following table of values

x	$y = \ln x$
2	.69315
2.5	.91629
3.0	1.09861

(32) Define maxima and minima of a tabulated function.

(33) Discuss General Quadrature Formula for Equidistant ordinates

(34) Establish Trapezoidal rule for ~~...~~ numerical integration

(34) obtain Simpson's one Third rule for numerical ~~integ~~ integration.

(35) obtain Simpson's three ~~eights~~ eights' rule for numerical integration.

(30) Obtain Weddle's Rule for numerical integration.

(31) Show that

$$\int_0^1 \frac{dx}{x} = \log 2 = .69315$$

(32) Obtain Homogeneous linear difference equation with constant coefficients

(33) Solve

$$2y_{k+2} - 5y_{k+1} + 2y_k = 0$$

(34) Solve

$$y_{k+1} - 2y_k = k+1$$

(35) Define curve fitting. Discuss the principle of least square for curve fitting.

(36) Discuss the method of changing origin and scale for simplifying the calculations in the case of curve fitting.

(37) Show that the linear difference equation of order n is given by

$$f_0(k)\, y_{k+n} + f_1(k)\, y_{k+n-1} + \dots + f_n(k)\, y_k = g(k)$$

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Subject:- Mathematics
Paper – 10th
Model Paper (Full Marks – 80)

Functional Analysis

(I) Define normed linear space with an example

(II) Define Banach space with an example.

(III) Construct a normed linear space which is not a Banach space

(IV) Prove that in a normed linear space E,

$$\big|\,\|x\|-\|y\|\,\big| \leq \|x-y\|$$

(V) Prove that in a normed linear space vector addition, norm and scalar multiplication are continuous function

(VI) Let T be a linear transformation of a normed linear space N into another linear space N' then show that the following statements are equivalent to one another.

(i) T is continuous

(ii) T is continuous at the origin

(iii) There exists a real number $K \geq 0$ such that

$$\|T(x)\| \leq K\|x\|, \forall x \in N$$

(iv) If $S = \{x : \|x\| \leq 1\}$ is the closed unit sphere in N then the image $T(S)$ is a bounded set in N'.

(7) If N and N' be normed linear spaces and let T be a linear transformation of N into N'. then the ~~inverse~~ inverse T^{-1} exists and is continuous on its domain of definition iff there exists a constant $m > 0$ such that

$$m\|x\| \leq \|T(x)\|, \forall x \in N$$

(8) Prove that for any normed linear space E, the identity transformation

$$I : E \to E \text{ defined by}$$

$$I(x) = x, \forall x \in E$$

is a continuous linear transformation

(9) Let N and N' be normed linear spaces with the same scalars and $B(N, N')$ the set of all bounded linear operator $T: N \to N'$, then $B(N, N')$ is a normed linear space with respect to the pointwise linear operations and the norm defined by

$$\|T\| = \sup_{\|f\| \leq 1} \|T(f)\|$$

Further if N' is a Banach space then $B(N, N')$ is also Banach space.

(10) Let M be a closed linear subspace of a normed linear space N and let T be the natural mapping of N onto N/M defined by $T(f) = f + M$. Then T is a continuous linear transformation for which ~~[illegible]~~ $\|T\| \leq 1$.

(11) Every n-dimensional linear space $L(F)$ is isomorphic to F^n.

(12) Define Dual space of a linear space.

(12) Show that every finite dimensional linear space is algebraically reflexive.

(14)

Banach space $C[a,b]$

(15) Let N and N' be normed linear space over the same scalar field and let T be a linear transformation of N into N'. Then T is bounded iff it is continuous

(16) Let N be a non-zero normed linear space and let

$S = \{x \in N : \|x\| \leq 1\}$ be a linear space of N.

Prove that

N is a Banach space $\Longleftrightarrow$ S is Complete

(17) Define equivalence Norm

(18) Let a linear space E be made into a normed linear space in two ways and let the two norms of a vector x be denoted by $\|x\|$ and $\|x\|'$. Show that the two norms generate the same topology on E iff $\exists$ two positive numbers m and M s.t.

$m\|x\| \leq \|x\|' \leq M\|x\|, \quad \forall x \in E$

(18) State and prove Hahn – Banach theo

(19) State and prove Open mapping theorem

(20) State and prove closed graph theorem

(21) State and prove Banach – Steinhaus theorem

or

The Principle of Uniform boundedness.

(22) Define Inner product space and Hilbert space with suitable example

(23) State and prove Schwarz inequality

(24) Prove that in a Hilbert space the inner product is jointly continuous

(25) If x and y are any two vectors in a Hilbert space then

(i) $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

(ii) $4(x,y) = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$

(26) If B a complex Banach space whose norm obeys the parallelogram law and if an inner product is defined on B by

$$4(x,y) = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2,$$ then B is a Hilbert space

(27) Define convex set. A closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

(28) State and prove Cauchy-Schwarz-Buniakowsky inequality

(29) Every finite dimensional linear inner product space is separable. Prove this

(30) State and prove Pythagorean theorem

(31) Define orthogonal complement.

(32) If M is a linear subspace of a Hilbert space H, show that M is closed $\Leftrightarrow M = M^{\perp\perp}$

(33) State and prove the projection theorem on a Hilbert space.

(34) The orthogonal complement of a subspace is itself a subspace.

(35) Define orthonormal set

(36) State and prove Bessel's inequality

(37) Show that two Hilbert spaces having the same dimension are isomorphic

(38) Prove the
$\|f\| - \|g\| \le \|f-g\|$

(39) Define complete orthonormal set

(40) State and prove Gram Schmidt ~~orthogo~~ orthogonalization process in a Hilbert space

(41) Prove that an orthonormal set in a Hilbert space is linearly independent

(42) State and prove the Re Riesz-Representation theorem for continuous linear functions on a Hilbert space.

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Subject:- Mathematics
Paper – 11th
Model Paper (Full Marks – 80)

Partial differential Equation

(1) Discuss Lagranges method ~~too~~ of solving the quasi linear partial differential equation of order one

$$P(x,y,z)\frac{\partial z}{\partial x}+Q(x,y,z)\frac{\partial z}{\partial y}=R(x,y,z)$$

(2) Find the condition for the equations

$$f(x,y,z,p,q)=0$$
or $g(x,y,z,p,q)=0$
to be compatible.

(3) Solve

$$p\tan x+q\tan y=\tan z$$

(4) Solve

$$y^2p-xyq=x(z-2y)$$

(5) Solve

$$xzp+yzq=xy$$

(6) Along every strip of the P.D.E

$F(x,y,z,p,q)=0$ the function
$F(x,y,z,p,q)$ is constant. Prove this

(7) Find the characteristics
of the P.D.E
$p^2+q^2=2$
and determine the integral
surface which pass through
$x=0, z=y$.

(8) Find the complete integral
of $(p^2+q^2)y = qz$

(9) Define surface orthogonal
to a given system of surfaces.

(10) Define compatible system
of first order equation

(11) Discuss the charpit's method
for solution of non-linear
P.D.E of first order.

(12) Define Jacobi method for
solution of non-linear P.D.E
of order one with more than
three independent variable

(13) Explain Cauchy's method
of characteristic

(14) Discuss method of finding the complementary function of the linear homogeneous P.D.E with constant coefficients

(15) Solve

$$\left(D^2+3DD'+2D'^2\right)z = x+y$$

(16) Solve

$$\left(2D^2-5DD'+2D'^2\right)z = 5\sin(2x+y)$$

(17) classify the linear P.D.E of second order and reduce it into canonical forms and discuss briefly the elliptic, parabolic and hyperbolic form

(18) Find the characteristic of the equation

$$u_{xx}+2u_{xy}+\sin^2(x)u_{yy}+u_y=0$$

when it is of hyperbolic type.

(19) classify and transform the following equation to a canonical form

(20) Transform the following differential equation to a canonical form

$$u_{xx}+2u_{xy}+4u_{yy}+2u_x+3u_y=0$$

(21) Reduce

$$\frac{\partial^2 z}{\partial x^2} = x^2\left(\frac{\partial^2 z}{\partial y^2}\right) \text{ to}$$

Canonical form

(22) write the Transport Equation for a linear hyperbolic system.

(23) Discuss derivation of one dimensional wave equation

(24) Discuss derivation of two dimensional wave equation

(25) Discuss derivation of one-dimensional ~~to~~ heat equation.

(26) Explain the Laplace's equation in spherical coordinates

(27) Define Harmonic function. If a harmonic function vanishes everywhere on the boundary, then it is identically zero everywhere.

(28) Discuss the separation of variables

(29) Find the general solution of one-dimensional wave equation

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{1}{c^2}\right)\left(\frac{\partial^2 y}{\partial t^2}\right)$$

(30) Discuss D' Alembert solution of one dimensional wave equation

(31) Obtain Green's function for the wave equation.

(32) Determine the Green's function for the Helmholtz equation for the half space $z \geq 0$.

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Subject:- Mathematics
Paper – 12^{th}
Model Paper (Full Marks – 80)

Paper – XII
Analytical Dynamics

1. Define the following terms
   (a) Generalised coordinates
   (b) degree of freedom
   (c) Holonomic system and Non-Holonomic system
   (d) Conservative and Non-Conservative system.
2. Derive Lagrange's equation of motion of a system of particle with given generalised coordinates.
3. Set up the Lagrangian for a simple pendulum and obtain an equation describing the motion.
4. Obtain the Lagrangian equation of motion for a double pendulum vibrating plane.
5. Show that the K.E is a holonomic dynamical system is a quadratic function of velocities.

(6) Deduce the principle of energy from the Lagrange's equation (conservative field)

(7) Use Lagrange's equation to find differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis

(8) Obtain the Lagrange's equation with impulsive forces

(9) Obtain the Lagrange's equations for Non-Holonomic system with ~~&~~ moving constraints.

(10) A particle of mass m moves under the influence of gravity on the inner surface of the paraboloid of revolution $x^2+y^2=az$ which is assumed frictionless. Obtain the equations of motion.

(11) Define phase space. Obtain Hamilton's canonical equation.

(12) If the Hamiltonian H is independent of t explicitly prove that it is

(a) a constant and is

(b) Equal to the total energy of the system

(13) State and prove Brachistochrone problem

(14) Discuss the Hamilton's variational principle

(15) obtain derivation of Hamilton's equations from the variational principle

(16) Discuss the principle of least action.

(17) write Distinction between Hamilton's principle and the principle of least action.

(18) use the variational method to show that the shortest curve joining two fixed points is a straight line.

(19) Prove that the shortest distance between two points on the surface of the sphere is the arc of the great circle connecting them.

(20) obtain Lagrange's equation from Hamilton's principle.

(21) Write the Hamiltonian and equation of motion for a simple pendulum.

(22) obtain Eulers equation from Hamilton's equation.

(23) Use Hamilton's equations find the equations of motion of a projectile in space.

(24) Use the variational method to show that the shortest curve joining two fixed points is a straight line.

(25) Discuss the stationary property of normal modes in the theory of small oscillation.

(25) Define point and canonical transformation and obtain the transformation equations

(26) obtain the conditions that a transformation from (q, p) basis to (Q, P) basis be canonical.

(27) Define Lagrange's bracket. Also show that invariance of Lagranges Bracket as a condition for canonical transformation

(28) Define Poisson Bracket. Show that the Poisson Bracket of two dynamical variable u and v under canonical transformation is invariant

(29) Write relation between Lagranges and Poisson Brackets

(30) Obtain Hamilton - Jacobi equation for a holonomic system given by the Canonical equations

(31) Obtain Hamilton - Jacobi equation for Hamilton's characteristic function

(32) Explain how Hamilton - Jacobi equation can be used to solve Kepler's problem for a particle in an inverse square central force field.

(33) Discuss Harmonic oscillator problem as an example of the Hamilton Jacobi method

(34) Obtain the Hamiltonian and Hamilton's equation for a charged particle, in an electromagnetic field.

(35) A particle of mass m moves in a force field of potential V

(a) write the Hamiltonian and

b) Hamilton's equation in Cartesian Coordinates.

(36) Use Hamiltonian equation to find the equation of motion of a projectile in space

(37) Find the equation of motion of a simple pendulum

(38) Discuss the simple Harmonic oscillator

(39) Obtain K.E of rigid body with a fixed point.

(40) Obtain Euler's equation from Lagrange equation

(41) Prove that the contact transformation possesses the group property. Also define subgroups of Mathieu transformation and extended point transformation.