Babasaheb Bhimrao Ambedkar Bihar University, Muzaffarpur
Directorate of Distance Education
P.G. 2nd Semester Examination 2016 (Session 2015-17)
Subject:- Mathematics
Paper – 5th
Model Paper (Full Marks – 70)

Answer any four questions:-
किन्हीं चार प्रश्नों के उत्तर दें।

1st Unit (Linear Algebra Projection mappings on a linear space)

1. Define a projection on a linear space show that a linear transformation E on V is a projection on some subspace if and only if it is idempotent.
2. Define a projection on a linear space and prove that a linear transformation T of a linear space L onto itself is a projection on L if and only if $L=M\oplus N$, where M and N are the range and the null spaces respectively.
3. Define range of a projection. Prove that two projections E and F have the same range if an only if EF = F and FE=E.
4. State and prove fundamentals theorem on homomorphism on linear space.

2nd Unit (Metric Vector Space)

5. Define bilinear from and symmetric bilinear form. If V is a finite dimensional vector space, then a bilinear form f and V is symmetric if and only if its matrix A in some (or every) ordered basis is symmetric, i.e. A^1. Prove this.
6. State and prove Sylvester's Theorem.
7. Define skew-symmetric bilinear form. Prove that every bilinear form on the vector space V over a subfield F of the complex numbers can be uniquely expressed as the sum of symmetric and skew-symmetric bilinear forms.
8. Prove that alternating bilinear form is skew-symmetric. But the converse is not, in general, true.

3rd Unit (Lattice Theory)

9. Define lattice as an order-structure and also as an algebraic structure and establish their equivalence.
10. Write a note on the principle of duality. If a and b are any two elements of a lattice such that $a \geq b$, prove that $\{x : a \geq x \geq b\}$ is a sub-lattice.
11. Prove that every chain is a distributive lattice.
12. Define modular lattice. State and prove the characterization theorem for modular lattice.

4^{th} Unit (Boolean Algebra)

13. Prove that a Boolean algebra is a complemented distributive lattice.
14. Prove that a complemented distributive lattice is a Boolean algebra.
15. Define Boolean algebra. Prove that no Boolean algebra can have three distinct elements.
16. Prove the every Boolean ring can be converted into a Boolean algebra.

5^{th} Unit (Canonical Forms)

17. Define similarly of linear transformations. Show that the relation of similarity is an equivalence relation in the set of all linear transformations on a vector space V(F).
18. If T and S are linear maps on a vector space V(F), the prove that T^2 is similar to S^2. Also show that T^{-1} is similar to S^{-1}, if T and S are invertible.
19. Let a be n x n matrix with all Eigen values zero. Show that A is nilpotent. Is the converse true? Or
    Show that a non-zero matrix is nilpotent if and only if all its Eigen values are equal to zero.
20. Prove that there exists an invertible matrix such that $P^{-1}AP$ is in Jordan canonical form.

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P.G. 2nd Semester Examination 2016 (Session 2015-17)
Subject:- Mathematics
Paper – 6th
Model Paper (Full Marks – 70)

Answer any four questions:-
किन्हीं चार प्रश्नों के उत्तर दें।

1st Unit (Analysis Functions)

1. Define analytic function and obtain Cauchy-Riem-Riemann condition for a function f(z) to be analytic.
2. Obtain Cauchy-Riemann equations for en analytic function.
3. Define Analytic and Regular functions. State and prove that necessary and sufficient conditions for f(z) to be analytic.
4. Find the analytic function of which x^3-$3xy^2$ is read part.

2nd Unit (Bilinear Transformations)

5. Define conformal transformation. Prove that the resultant of two bilinear transformations is a bilinear transformation.
6. Define bilinear transformation. Show that cross-ration remains invariant under a bilinear transformation.
7. Show that bilinear transformation transforms circles intro circles.
8. Define resultant of two bilinear transformations. Show that every bilinear transformation can be expressed as the resultant of an even number of transformations.

3rd Unit (Complex Integration)

9. State and prove Cauchy's fundamental theorem.
10. Derive Cauchy integrate formula for the derivative of an analytic function.
11. State and prove Morera's theorem.
12. Establish Cauchy's inequality.

4th Unit (Residue Theory)

13. Define pole and singularity, and discuss different kinds of singularity times.
14. Prove that zeros and poles of an analytic function f(z) are isolated.
15. State and prove Principle of Argument.
16. State and prove fundamental theorem of algebra.

5th Unit (Applications)

17. Discuss uniqueness of solutions of Dirichlet's problem.
18. State Green's function. Discuss its existence and uniqueness.
19. If Green's function for I(c) is symmetric, i.e., if a and b are different points in I(c), then show that G(a,b) = G(b,a).
20. If f(z) is analytic in a domain R and f(z) = 0 at all points of an arc PQ inside R, then show that f(z) = 0 throughout R.

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P.G. 2nd Semester Examination 2016 (Session 2015-17)
Subject:- Mathematics
Paper – 7th
Model Paper (Full Marks – 70)

Answer any four questions:-
किन्हीं चार प्रश्नों के उत्तर दें।

1st Unit (Existence Method)

1. State and prove the existence theorem for the initial value problem.
   $dy/dx = f(x,y)$; $f(x_0) = y_0$
2. State the conditions under which the initial value problem.
   $dy/dx = f(x,y)$; $f(x_0) = y_0$
   has at least one solution. Show that the solution is unique, if it exists.
3. Define initial value problem and find its equivalent integral equation.
4. Discuss Picard's method for the solution of initial value problem of ordinary differential equation. Or
   Discuss Picard's method for numerical solution of ordinary differential equation of first order.

2nd Unit (Basic Theorems)

5. State and prove Peano's Existence theorem and its corollary.
6. State and prove Picard's lindal of theorem.
7. State and prove Gron Wall's integral inequality.
8. State and prove Ascohi-Arzela theorem.

3rd Unit (System of Differential Equations)

9. Discuss the method of solving a system of ordinary differential equations by finding integrable combinations.
10. Discuss the method of integration of differential equations by reducing it to a single equation of higher order.
11. Solve the following reducing equations into one equation of higher order.
    (i) $dx/dt = y$, $dy/dt = x$
    (ii) $dx/dt = 3x-2y$, $dy/dt = 2x-y$
12. solve
    (i) $dx/dt - 7x + y = 0$, $dy/dt - 2x - 5y = 0$
    (ii) $dx/dt = -5x - 2y$, $dy/ = x - 7y$, $x(0) = 2$, $y(0) = 1$

4^{th} Unit (Stability)

13. Show that the stability problem can be reduced to be stability of null or trivial solution of a system of differential equation. Also discuss stability of autonomous system and stability of the rest point.
14. Show that each solution of the equation dx/dt + x = 0 is asymptotically stable.
15. Examine the stability of the system whose equation for perturbed motion is given by dx/dt = - α y & dy/dx = α x.
16. Examine the stability of the null solution of the equation of the perturbed motion is (i) dx/dt = y and dy/dt = x.

5^{th} Unit (Polynomials)

17. Define Hermite differential equation and find its solution.
18. Prove the following recurrence relation.
    (i) $H^1{}_n{}^{(x)} = 2\, n\, H_{n-1}{}^{(x)}$
    (ii) $H_{n+1}{}^{(x)} = 2\, xH_n(x)$
19. Prove the following:-
    (i) $(n+1)\, L_{n+1}{}^{(x)} = (2n+1-x)\, L_n{}^{(x)} - nL_n - 1^{(x)}$
    (ii) $xL^1{}_n{}^{(x)} = nLn^{(x)} – nL_{n-1}{}^{(x)}$
20. Prove that $J_n(x)$ is the coefficient of z^n in the expansion of $e^x(z-1/z)/2$ in ascending or descending power of z, when n is a positive integer. Also prove that $J_n(x)$ is the coefficient of z^n multiplied by $(-1)^n$ in the expansion of above expression.

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Answer any four questions:-
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(Set Theory)

1. Define the cardinal equivalence of two sets. Prove that the relation of cardinal equivalence is an equivalence relation on $P(\Omega)$, where $P(\Omega)$ denotes the power set of the universal set Ω.
2. Prove that:-.
   (i) Union of finite number of denumerable sets is denumerable.
   (ii) Union of a countable infinite number of denumerable sets is denumerable.
   (iii) Cartesian product of two denumerable sets is denumerable.
3. If a denumerable is subtracted from a non-denumerable set, the remaining set is non-denumerable.
4. If X is any set, then prove that
   Card $P(X) = 2^{\text{Card } X}$, P(X) being the power set of X.
5. If m and n are any two cardinal numbers, then prove that one and only one of the following three cases hold:
   (i) $M < n$ (ii) $n < m$ (iii) $m = n$
6. State and prove well-ordering theorem.

(Graph Theory)

7. What do you mean by a Graph? Discuss its utilities
   (i) Discuss Konigsberg Bridge Problem.
   (ii) Discuss graphs representing problem of supplying water, gas and electricity in each of three given houses.
8. Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can this arrangement last?
9. Show that the maximum number of edges in a simple graph with n vertices is $n(n-1)½$.
10. Define and explain path, circuit, cycle sub-graph of a graph with suitable examples.
11. State and prove Euler's equation for a connected planner graph.
12. Discuss planar graph. Prove that the complete graph of five vertices is non-planar.
13. Write a short note on tree.
14. Prove that a tree with n vertices has (n-1) edges.
15. Prove that a connected planar graph with n vertices and e edges has e-n+2 regions.