Directorate of Distance Education

(B.R.A. Bihar University, Muzaffarpur)

P.G. 1st Semester Examination 2016

January Session 2016-18

Sub:- Mathematics (Paper 1)

Model Papers

(1) Define composition series of a group with an example.

(2) Prove that there exists at least one composition series for every finite group G.

(3) State and prove Jordan Holders theorem.

(4) (a) Show that a finite p-group is nilpotent

(b) Define ~~no~~ nilpotent grp.

(5) State and prove the factorisation theorem.

(6) Show that any finite extension of a field is an algebraic extension of the field.

(7) State and prove the ~~...~~ Galois fundamental theorem.

(8) Define Solvable group with an example.

(9) Define commutator sub-group with an example

(10) A group G is solvable iff $G^{(n)} = \{e\}$ for some integer n

(11) Every subgroup of solvable group is solvable.

(12) If H be a normal subgroup of a solvable group G then G/H is also solvable.

(13) Prove that product of solvable group is solvable.

(14) Prove that every abelian group is solvable

(15) Prove that every homomorphic image of a solvable group is solvable

(16) Define module. Unital R- module with an example

(17) Prove that every abelian group (G, +) is module over the ring of integers.

(18) Prove that the intersection of any two submodules A and B of a R- module M is also a submodule of M.

(19) Prove that Union of submodules is a

submodule iff one is contained in the other.

(20) Prove that the range of homomorphism of an R-module M is a submodule of N.

(21) If A and B are submodules of an R-module M then A+B is also a submodule of M.

(22) Prove that multiplicative group of Galois fields cyclic

(23) Let $f: M \to N$ is homomorphism then prove that

(a) $f(0) = 0$

(b) $f(-m) = -f(m)$ $\forall m \in M$

(25) Let M and N be R-module and $f: M \to N$ be homomorphism then the kernel of f is submodule of M.

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Sub:- Mathematics (Paper 2)

Model Papers

(1) State and prove Bolzano - Weierstrass theorem.

(2) Prove that a bounded monotonic function is a function of bounded variation.

(3) Prove that the sum and difference of two functions of bounded variation is also of bounded variation.

(4) State and prove Jordan's theorem

(5) State and prove Heine - Borel theorem

(6) Define the Rieman - Stieltjes integral

(7) Let f and g be any complex valued function of bounded variation on [a, b] then f+g and f.g are also of bounded variation. (Prove this)

(8) State and prove first mean value theorem

(9) Define power series

(10) State and prove Abel's theorem

(11) State and prove Tauber's theorem

(12) State and prove Inverse function theorem

(13) Define Jacobian.

(14) If $u_1 = \frac{x_2 x_3}{x_1}$, $u_2 = \frac{x_1 x_3}{x_2}$, $u_3 = \frac{x_1 x_2}{x_3}$ prove that $J(u_1, u_2, u_3) = 4$

(15) If
$u = x^2+y^2+z^2$,
$v = x+y+z$
$w = xy+yz+zx$
then show that the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ vanishes identical.

(16) If $x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

then find $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

(17) If $u^2+v^2 = x+y$, $u^3+v^3 = x^3+y^3$ then find

$$\frac{\partial(u,v)}{\partial(x,y)}.$$

(18) Prove that the power series $\sum a_n z^n$ is either

(i) convergent for all values of z

(ii) converges only for $z=0$

(iii) converges for z in some region in the complex plane

(19) Find the radius of convergence of the series

$$\sum \frac{n^n x^n}{\lfloor n}$$

(20) Prove that a bounded monotone function is a function of bounded variation

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Sub:- Mathematics (Paper 3)

Model Papers

(1) Define outer-measure

(2) Let A and B be two sets such that $A \subseteq B$. Then show that
$$m^*(A) \leq m^*(B)$$

(3) Define Cantor Ternary set and show that Cantor ternary set is measurable and its measure is zero.

(4) Prove that outer measure is translational invariant.

(5) Define measurable function.

(6) Define characteristic function. Prove that characteristic function of a set A is measurable iff A is measurable.

(7) Give four equivalent definition for the Lebesgue measurability of a

real valued function.
Prove their equivalence

(8) Let f and g be measurable function defined over a measurable set E. Show that $f+g$, $f-g$, $f.g$ are measurable functions over E.

(9) A continuous function defined over a measurable set E is measurable. Is the converse of the theorem true?

(10) Show that every bounded measurable function f defined on $[a,b]$ is Lebesgue-integrable over $[a,b]$

(11) Let f and g be bounded function in $L[a,b]$ then $f+g \in L[a,b]$ and

$$\int_a^b (f+g) = \int_a^b f + \int_a^b g$$

(12) State and prove Riesz theorem

(13) State and prove Egoroff's theorem

(14) State and prove Lebesgue monotone convergence theorem.

(15) State and prove Lusin theorem.

(16) State and prove Jordan decomposition theorem

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Sub:- Mathematics (Paper 4)

Model Papers

(1) Define pological space.
List all pologies on $\{x, y, z\}$

(2) State and prove Kuratowski closure axioms.

(3) Define accumulation point. Show that an adherence point need not be an accumulation point.

(4) Show that f is continuous iff inverse image of every open set is open

(5) Show that f is continuous iff inverse image of every closed set is closed

(6) Let X and Y be two topological spaces and f is a mapping of X into Y. A necessary and sufficient condition for f to be continuous is that for each subset A of X
$f(\bar{A}) \subseteq \overline{f(A)}$

(7) Define homeomorphism. State and prove fundamental theorem for homeomorphism

(8) Prove that every finite ~~two~~ Topological space is compact

(9) Prove that every closed subset of a compact space is compact

(10) Prove that every compact subset of Hausdorff space is closed.

(11) Construct a topological space which is compact but not connected .

(12) Define T_0, T_1, T_2, T_3, T_4 space.

(13) Example of a topological space which is T_2-space but not a T_1-space

(14) give an example of a topological space which is T_1- space but not a T_2-space

(15) Prove that every compact Hausdorff space is normal.

(16) State and prove Urysohn Lemma

(17) A topological space X is a T_0-space iff $x, y \in X$, $x \neq y$ $\Rightarrow \overline{\{x\}} \neq \overline{\{y\}}$

(18) Prove that a property being a T_0-space is topological.

(19) Show that every T_3-space is T_2-space.

(20) Define (i) Adherent point (ii) closure (iii) Interior (iv) boundary

(21) (i) Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$