

Directorate of Distance Education

(B.R.A. Bihar University, Muzaffarpur)

T.D.C. 2nd Semester Examination

(Session 2012-13)

Sub:- Mathematics (Hons.) (Paper - II)

Model Papers

1. State and prove De Moivre's theorem.
2. Find the expansion of $\sin x$ in ascending powers of x .
3. Find the expansion of $\cos x$ in ascending powers of x .
4. State and prove Lagrange's theorem.
5. Find the sum of sines of n angles in A.P.
6. Find the sum of cosines of n angles in A.P.
7. Solve $n^2 - 1 = 0$
8. If $\tan x = n \tan y$ find a series for x .
9. Prove that n th roots of unity forms a series in A.P.
10. Solve $n^5 - 1 = 0$
11. If $(1+x)^n = p_0 + p_1x + p_2x^2 + \dots$
then prove that
$$p_0 - p_2 + p_4 - \dots = 2^{\frac{n}{2}} \cos \frac{n\pi}{4}$$

and
$$p_1 - p_3 + p_5 - \dots = 2^{\frac{n}{2}} \sin \frac{n\pi}{4}$$

(12) Prove that
 $(1-i)^n + (1+i)^n = 2^{n/2} \cos \frac{n\pi}{4}$

(13) If $z = \cos\left(\frac{\pi}{2^x}\right) + i \sin\left(\frac{\pi}{2^x}\right)$
 then prove that

$$z_1 \cdot z_2 \cdot z_3 \dots = -1$$

(14) If $(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = A + iB$
 then prove that

(i) $\tan^{-1} \frac{b_1}{a_1} + \tan^{-1} \frac{b_2}{a_2} + \dots + \tan^{-1} \frac{b_n}{a_n} = \tan^{-1} \frac{B}{A}$

(ii) $(a_1^2 + b_1^2)(a_2^2 + b_2^2) \dots (a_n^2 + b_n^2) = A^2 + B^2$

(15) Expand $(1 - 2x \cos \alpha + x^2)$ in a series of powers of x when $|x| < 1$

(16) Using De Moivre's theorem solve $x^7 + x^5 + x^2 + 1 = 0$

(17) Sum the series
 $1 + \cos \alpha + e^{-2} \cos 2\alpha + e^{-4} \cos 3\alpha + \dots$

(18) Sum the series
 $\sin \alpha + \frac{1}{2} \sin 2\alpha + \frac{1}{2^2} \sin 3\alpha + \dots$

(19) Sum the series
 $1 + \frac{1}{2} \cos 2\alpha + \frac{1}{2 \cdot 4} \cos 4\alpha + \frac{1}{2 \cdot 4 \cdot 6} \cos 6\alpha + \dots$
 where $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

20) Sum the Dens

$$1 + \frac{C_1 40}{C_2} + \frac{C_1 80}{L} \quad \text{---}$$

L.P.P

- 1) Explain general and matrix formulation of L.P.P
- 2) Define convex set. Prove that intersection of two convex sets is a convex set.
- 3) Prove that a hyperplane is a convex set.
- 4) Prove that a sphere is a convex set.
- 5) Prove that the set of feasible solutions of L.P.P is a convex set.

6) Solve graphically the following L.P.P

maximize $Z = 3x_1 + 5x_2$

Subject to

$$x_1 + 2x_2 \leq 2000$$

$$x_1 + x_2 \leq 1500$$

$$x_2 \leq 600$$

$$x_1 \geq 0, x_2 \geq 0$$

② Solve graphically the following L.P.P

Minimise $z = x_1 + x_2$

Subject to Constraints

$$x_1 + 2x_2 \leq 20$$

$$x_1 + x_2 \leq 15$$

$$x_1 \leq 6$$

$$x_1, x_2 \geq 0$$

Theory of Equations

① State and prove fundamental theorem of algebra

② Find the condition that the roots of the equation

$$x^3 + px^2 + qx + r = 0$$

should be in H.P.

③ Find the condition that the roots of the equation of the biquadratic

$$x^4 + px^3 + qx^2 + rx + s = 0$$

may be in G.P.

④ If α, β and γ be the roots of the equation $x^3 + px^2 + qx + r = 0$

then calculate

$$(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$$

⑤ If α, β, γ be the roots of the cubic $x^3 + px^2 + qx + r = 0$, find the value of the symmetric function $\sum \frac{\beta + \gamma}{\alpha} + \frac{\gamma + \alpha}{\beta} + \frac{\alpha + \beta}{\gamma}$

⑥ If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ find the value of

$$\sum \frac{\alpha^2 + \beta^2}{\alpha + \beta}.$$

⑦ Solve the equation

$x^2 - 7x + 36 = 0$ given that one root is double another root.

⑧ Prove that in equations with real coefficients irrational roots occur conjugate pairs.

⑨ Prove that every equation of n th degree has n roots and no more.

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Sub:- Mathematics (Subsidiary & General) (Paper - II)

Model Papers

- (i) State and prove De Moivre's theorem
- (ii) Find the expansion of $\sin x$ in ascending powers of x
- (iii) Find the expansion of $\cos x$ in ascending powers of x
- (iv) State and prove Gregory's theorem
- (v) Expand $\cos^{-1}(a+ib)$ in the form of $A+iB$
- (vi) If $\tan x = n \tan y$ then find a series for x .
- (vii) Solve $x^5 - 1 = 0$
- (viii) Expand $\tan^{-1}(a+ib)$ in the form of $A+iB$
- (ix) If $x_1 = \cos\left(\frac{\pi}{2r}\right) + i \sin\left(\frac{\pi}{2r}\right)$ then prove that $x_1 \cdot x_2 \cdot x_3 \dots = -1$

(10) Solve $x^4 - 1 \Rightarrow$

~~(11)~~ Real Analysis

- (i) State and prove Cauchy's general principle of convergence
- (ii) Prove that every convergent sequence is bounded.
- (iii) State and prove Cauchy's root test
- (iv) D'Alembert's test
- (v) State and prove ratio test
- (vi) Define continuity and differentiability of a function
- (vii) Prove that a function which is differentiable at a point must be continuous at that point. But the converse is not true.
- (viii) Show that the sequence (a_n) is not convergent where $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$.

9) Establish the existence of the limit in the sequence (a_n) defined by $a_n = \sqrt[n]{n}$ and find the limit

10) Test the series

$$\frac{1}{1 \cdot 2 \cdot 5} + \frac{3}{2 \cdot 5 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$$

11) Test whether the series whose general term is given by

$$\sqrt{x^2+1} - \sqrt{x^2-1}$$

12) Examine the continuity and differentiability of the function defined by

$$f(x) = x^2 \sin \frac{1}{x}, \quad x \neq 0$$

$$f(0) = 0$$

at $x=0$

Analytical Geometry of two dimensions.

(i) Discuss the conditions for orthogonal intersection of two circles

(ii) standard equation ellipse, parabola, Hyperbola.

(iii) Find the condition that the line $lx + my + n = 0$ may touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Analytical geometry of Three dimensions :-

- ① If l, m, n be the direction cosine of a line prove that $l^2 + m^2 + n^2 = 1$
- ② Find the equation of a plane passing through three given non-collinear points
- ③ Find the angle between two straight lines whose direction cosines are given
- ④ Prove that every first-degree equation in x, y, z represents a plane
- ⑤ Find the equation of a plane in normal form
- ⑥ Find the equation of the plane passing through the point $(2, -1, 4)$ and perpendicular to the join of $(1, -2, 5)$ and $(2, 1, 3)$
- ⑦ Find the equation of the plane in intercept form.
- ⑧ Prove that the straight lines whose direction cosines are given by

$$2l + 2m - n = 0 \text{ and}$$

$$m^2 + nl + lm = 0$$