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Directorate of Distance Education
T.D.C. 1st Semester Examination 2016
January Session 2016-19
Sub:- MATHEMATICS (Hons.)
Model Paper (Paper: 1) Full Marks 80

(1) $A \cup (B \times C) = (A \cup B) \times (A \cup C)$

(2) $A \cap (B \times C) = (A \cap B) \times (A \cap C)$

(3) $(A - B) \times C = A \times C - B \times C$

(4) If $A \subseteq B$ then prove that

$A \times A = (A \times B) \cap (B \times A)$

(5) Define relation with an example

(6) Define reflexive, symmetric and transitive relation

(7) Define an equivalence relation

(8) State and prove Fundamental theorem on equivalence relation

(9) Prove that two equivalence classes are either disjoint or identical

(10) Define Partial order relation. Give an example of a partial order relation which is also a ~~tot~~ total order relation

(11) Define countable and uncountable sets

(12) Prove that set of real numbers is uncountable

(13) Prove that denumerable union of denumerable sets is denumerable

(14) Prove that the set of algebraic numbers is denumerable

(15) Prove that the set $[0, 1]$ is non-denumerable

(16) Prove that cartesian product of two denumerable sets is denumerable

(17) If R be the relation on Z defined by $|a-b| \leq \frac{1}{2}$ then is R an equivalence relation?

Matrix

(1) If A, B, C are three matrices can formable for addition and Multiplication then prove that

(a) $A \cdot (B + C) = A \cdot B + A \cdot C$

(b) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

(2) Prove that $(AB)' = B'A'$

(3) Find the adjoint and inverse of

$$\begin{bmatrix} 0 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & \end{bmatrix}$$

(4) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

(5) Show that the matrix

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

is orthogonal.

(6) Prove that

$A(\operatorname{adj} A) = (\operatorname{adj} A)\, A = |A|\, I_n$

(7) Define

(a) Symmetric matrix
(b) Skew Symmetric matrix
(c) Hermitian matrix
(d) Skew-Hermitian matrix

(8) Show that every square matrix is uniquely expressible as the sum of a Hermitian and a skew-Hermitian matrix

(5) Test the consistency and solve the equations

$x - 3y - 8z = -10$
$3x + y - 4z = 0$
$2x + 5y + 6z = 13$

(6) ~~Sol~~ Show that
Inverse of orthogonal matrix is orthogonal
(7) Prove that transposed orthogonal matrix is orthogonal

(8)
L.P.P

(1) Give the main formulation of LPP.
(2) Show that a Hyperplane is a convex set
(3) Show that a sphere is a convex set
(4) Show that set of all feasible solution of L.P.P is a convex set.
(5) Prove that intersection of two convex sets is convex set.
(6) Solve graphically the following L.P.P

Max $Z = 3x_1 + 5x_2$

Subject to

$x_1 + 2x_2 \le 2000$
$x_1 + x_2 \le 1500$
$x_2 \le 600$

& x_1 and $x_2 \ge 0$

(7) Solve by Simplex method the following LPP

Maximize $Z = 5x + 3y$

Subject to

$x + y \le 2$
$5x + 2y \le 10$
$3x + 8y \le 12$
$x, y \ge 0$

Theory of equations

(1) Prove that in an equation with real coefficients irrational roots occur in conjugate pairs
(2) Prove that every equation of the nth degree has n roots and no more
(3) State and prove fundamental theorem of algebra
(4) Solve the equation $x^3 - 7x + 6 = 0$ given that one root is double another root.

(5) If α, β, γ be the roots of the cubic $x^3 - px^2 + qx + r = 0$ then evaluate

$(\alpha^2+1)(\beta^2+1)(\gamma^2+1)$

(7) Find the condition that the roots of the equation

$x^4 + px^3 + qx^2 + rx + s = 0$

may be in G.P

(8) Find the condition that roots of the equation

$x^4 + px^3 + qx^2 + rx + s = 0$

may be in G.P

(9) If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$ then evaluate $\sum \frac{\beta^2+\gamma^2}{\beta+\gamma}$

Trigonometry

(I) State and prove De Moivre's theorem

(II) Find the expansion of $\sin x$ in ascending powers of x

(III) Find the expansion $\cos x$ in ascending powers of x

(IV) State and prove Gregory's theorem

(V) Find the sum of sines of n angles in AP

(VI) Find the sum of cosines of n angles in AP

(VII) If $\tan x = \tan y$ find a series for x

(VIII) Find the sum to the following

$$\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{3}\sin 3\alpha + \cdots$$

(IX) Find the sum to the series

$$1 + \frac{1}{2}\cos\theta + \frac{1}{2\cdot 4}\cos 2\theta + \frac{1\cdot 3}{2\cdot 4\cdot 6}\cos 3\theta + \cdots = \alpha$$

where $-\pi < \theta < \pi$

(X) Express $\sin(x+iy)$ in terms of $A + iB$